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Mathematical Modeling and Cost Minimization in Functional Dairy Product Development: A Case Study on Fruit Yogurt

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Abstract: In recent years, the food industry has placed increasing emphasis on cost efficiency and nutritional quality, driven by consumer demand for affordable yet healthy products. Among dairy-based functional foods, fruit yogurt has become one of the most popular fermented products due to its balanced nutritional profile, sensory appeal, and versatility. However, developing an optimal formulation that maintains quality while minimizing production costs remains a key challenge in dairy process engineering.

• Introduction

Yogurt is usually produced as semisolid food products, which derived from processed animal milks under appropriate fermentation conditions with the involvement of *Streptococcus salivarius ssp thermophilus* and *Lactobacillus delbrueckii ssp bulgaricus* bacteria. The combination of nutritional components and live microorganisms forms the basis for a highly nutritious food that can easily complement a healthy dietary regimen.

• Material and method

According to Micula and Rotaru an optimization model consists of three parts as follows:

1. The variables of the model are positive real numbers to be determined. Let's have $x_1, x_2, \dots, x_n$. There are models where these variables are positive integers, and may represent for instance the livestock in a farm.
2. The constraints to the model are m equations and inequalities containing the variables $x_1, x_2, \dots, x_n$.

The right of each member is the resource limit restrictions addressed by restriction. This part of the mathematical model is also known as the system of restrictions.

Six main ingredients were considered in the optimization model: skimmed milk, whole milk (3.5% fat), sugar, fruit puree, starter culture and stabilizer.

• Results and discussions

For each ingredient, parameters were defined including protein content, fat content, total solids, cost per 100 g, and proportion in the final mixture (decision variable) (Table 1).

	A	B	C	D	E	F	G	H
		Protein (%) p_i	Fat (%) f_i	Total Solids (%) s_i	Cost (€/kg) c_i	Min (%)	Max (%)	Variable: Amount (%) x_i
1	Ingredient							
2	Skimmed milk	3.4	0.5	9	0.8	50	80	0
3	Whole milk	15	3.5	12	1	0	20	0
4	Sugar	0	0	100	0.6	5	10	0
5	Fruit puree	0.5	0.2	30	1.2	10	15	0
6	Stabilizer	0	0	90	3	0.2	0.5	0
7	Starter culture	4	0	10	5	0.05	0.1	0

Objective function:

Minimizing total cost per 100 g:

$$C = \sum_{i=1}^6 (c_i * x_i / 100),$$

where C represents total cost (€/100 g product), c_i the cost per kg of ingredient i , and x_i the fraction of ingredient i in the final mixture.

Subject to the following constraints (restriction 1)

$$\sum_{i=1}^6 x_i = 100 \text{ (total mass fixed at 100 g)}$$

$$\sum_{i=1}^6 (p_i * x_i / 100) \geq 3\% \text{ (total proteins)}$$

$$\sum_{i=1}^6 (f_i * x_i / 100) \rightarrow 1.5\% - 3\%$$

$$\sum_{i=1}^6 (s_i * x_i / 100) \geq 12$$

$$0 \leq x_i \leq x_{i,max} \text{ and } x_i \geq x_{i,min} \text{ (ingredient limits).}$$

The Simplex LP solving method was selected for linear optimization. (Table 2)

i		j		
Constraints	Formula_or_Value	Variable: Amount (%) x_i	Formula_or_Value	
Total cost per 100 g (I2)	=SUMPRODUCT(I2:I7;H2:H7)/100	60.4	Total cost per 100 g (I2)	0.87
Total mass fixed at 100 g	=SUM(H2:H7)	19	Total mass fixed at 100 g	100
Total proteins (%) (I4)	=SUMPRODUCT(B2:B7;H2:H7)/100	10	Total proteins (%) (I4)	5.011
Total fat (%) (I5)	=SUMPRODUCT(C2:C7;H2:H7)/100	10	Total fat (%) (I5)	1
Total solids (%) (I6)	=SUMPRODUCT(D2:D7;H2:H7)/100	0.2	Total solids (%) (I6)	20.94375
		0.05		

Conclusions

Mathematical modeling is a very powerful and flexible tool that finds applicability in many fields. This paper shows how the SOLVER tools of Excel software is used easily for complex optimization problems, specifically for optimizing cost function of fruit yogurt.

The integration of mathematical modeling and optimization fosters sustainable and competitive practices in the modern dairy industry